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Linear Compressibility of Ice

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A novel technique of measuring the linear compressibility of ice at relatively low pressures (<0.5 kb) is described. A cathetometer was used in conjunction with a window-equipped pressure chamber to measure changes in the lengths of ice specimens compressed hydrostatically to 0.31 kb. A mean linear compressibility of 3.7 Mb^{-1} was obtained at $-10°\text{C}$, and the compressibilities perpendicular and parallel to the c axis of single crystals of ice were found to agree within 10%.

Few accurate data exist on the direct determination of the compressibility of ice in the low-pressure region (<0.5 kb). The most precise determination of the volume compressibility of ice in this low-pressure region would seem to be that measured by *Richards and Speyers* [1914], who obtained a value of 12 Mb^{-1} at 7.03°C in the pressure range 0.1–0.5 kb. This value was 2–3 times smaller than that indicated by the experimental data of *Bridgman* [1912] over the same pressure range. However, compressibilities computed from measurements of the elastic constants of single-crystal ice [e.g., *Jona and Scherrer*, 1952; *Bass et al.*, 1957; *Dantl*, 1969] all tend to confirm the experimental determination of Richards and Speyers.

In this study we have measured directly the linear compressibility of several types of air-free, crack-free chemically pure ice. The technique used eliminates the need for jacketed samples, permits measurements of compressibility as a function of crystallographic orientation of the specimens, and does not require knowledge of either the compressibility of the pressure medium or the volume changes occurring during compression. Our data are believed to be the first direct measurements of the linear compressibility of ice.

EXPERIMENTAL METHODS

Details of the experimental setup are illustrated in Figure 1. Major components of the test apparatus included a reservoir tank with an auxiliary pump, a manually operated 11-cm^3 displacement pump, an accurately calibrated Heise pressure gage, and the pressure chamber. Water-saturated kerosene was used as the pressure medium. The pressure chamber comprised a standard 12.7-cm ID pressure vessel, which could be securely bolted at one end and was fitted at the other end with a 7.6-cm-thick pressure sight glass. A Beck 1-meter-range cathetometer fitted with a 2.5-cm-range micrometer was mounted in front of the pressure window to measure directly the change in length ΔL of a specimen of ice of length L_0 compressed hydrostatically from zero gage pressure to 0.31 kb (ΔP). Accordingly, we can express the linear compressibility θ as

$$\theta = -(1/L_0)(\Delta L/\Delta P) \qquad (1)$$

Absolute accuracy of the pressure reading at 0.31 kb is estimated at 1.8 bars, i.e., 0.55%. All measurements were conducted in a temperature-controlled cold room maintained at a temperature setting of $-10°\text{C}$. However, temperatures in the chamber were monitored with a thermocouple to ensure that the total temperature difference during tests did not exceed 0.3°C. It was calculated that the correction for thermal expansion (or contraction) could be ignored if the temperature difference was held to $<0.3°\text{C}$.

Measuring technique. The technique used to measure linear compressibility is illustrated in Figure 2. Prisms of ice measuring 8.0 cm × 1.0 cm × 1.5 cm were frozen carefully to the base of a T-shaped aluminum support. Thin strips of moderately stiff photographic paper were laid carefully across the stem of the T

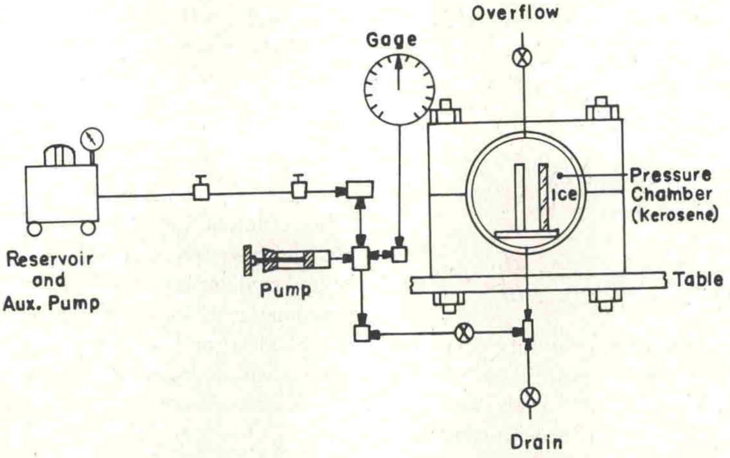

Fig. 1. Schematic drawing of compressibility apparatus. A cathetometer (not shown in diagram) is placed in front of the window-equipped chamber to measure the linear deformation of the compressed ice sample.

support and the ice specimen and frozen on with chilled degassed water from a syringe. As is illustrated in Figure 2*a,* each strip possessed a very sharp black-white border, which facilitated accurate focusing of the cathetometer cross hairs. Once attached, each marker strip was then sliced through obliquely with a sharp razor blade to permit each side of the severed marker strip to move freely during compression. Care was taken to ensure that the severed strips remained perfectly aligned after cutting. The alignment was checked before pressurizing the liquid in the chamber. No visible distortion of the marker strips was observed in any of the tests. As is indicated in Figure 2*a,* two marker strips were attached at the top, mainly as a safeguard against accidental detachment of the primary (topmost) marker but also as an additional check on the cathetometer measurements.

During compression both the specimen and the T support will undergo shortening. As is demonstrated in Figure 2*b,* this shortening will produce maximum offset of the marker strips at the top and negligible offset at the bottom. However, since the observed offset is for ice relative to aluminum, this offset will measure less than the true shortening ΔL of the ice by an amount equal to the shortening of the aluminum support. If we designate the observed offset as ΔL_1

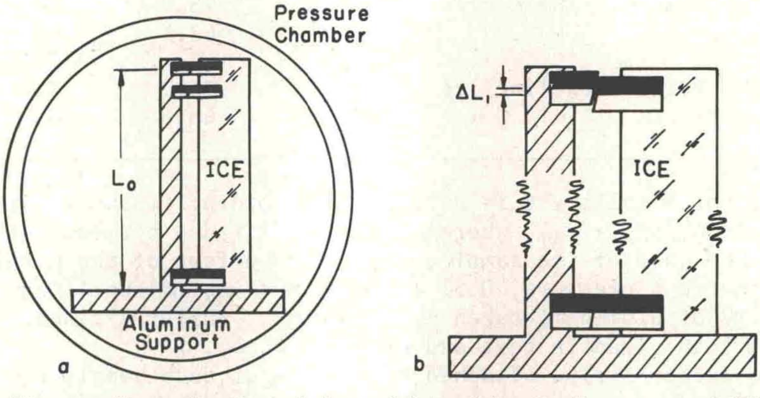

Fig. 2. Diagrams illustrating the technique of determining the linear compressibility of ice samples for (*a*) conditions at zero gage pressure and (*b*) conditions at test pressure. Equations relating strip marker offset ΔL_1 to compressibility of the ice sample are given in text.

and the shortening of the aluminum stem as ΔL_{Al}, then

$$\Delta L_1 = \Delta L - \Delta L_{Al}$$

or

$$\Delta L = \Delta L_1 + \Delta L_{Al}$$

Substituting in (1), we have

$$\theta = -\frac{1}{L_0}\frac{\Delta L_1 + \Delta L_{Al}}{\Delta P} = -\frac{1}{L_0}\frac{\Delta L_1}{\Delta P} + \theta_{Al} \qquad (2)$$

where θ_{Al} is the linear compressibility of aluminum. A search of the literature [e.g., *Birch*, 1966; *Bridgman*, 1923] showed some variation in the measured values of θ_{Al}. A compressibility of 0.43 Mb^{-1} was adopted here. This value is less than that deduced for ice on the basis of the volume compressibility data of *Richards and Speyers* [1914] by nearly an order of magnitude.

During tests several individual determinations of the marker offsets were made by using different initial settings of the cathetometer micrometer. Readings could generally be reproduced within the 5-μm readability of the micrometer, which has a stated accuracy of ± 2.5 μm. Calibration tests were made with an aluminum prism in place of an ice specimen. Zero offsets were observed for all positions of marker strips at the test pressure. The fact that all

TABLE 1. Linear Compressibility θ of Ice as a Function of Ice Type and Crystallographic Orientation

Test Sample*	Ice Type and Orientation	L_0, mm	ΔL_1, μm	θ, Mb^{-1}
1*a*	Laboratory single crystal ⊥c†	74.8	69	3.4
b		65.5	63	3.5
2*a*	Laboratory single crystal ⊥c	75.5	75	3.6
b		61.6	64	3.8
3*a*	Laboratory single crystal ⊥c	75.5	66	3.2
b		61.6	63	3.7
4*a*	Laboratory single crystal ||c	78.5	79	3.6
b		71.1	70	3.6
5*a*	Glacier single crystal ⊥c	69.3	72	3.8
b		58.5	63	3.9
6*a*	Glacier single crystal ⊥c	62.4	64	3.6
b		53.6	56	3.8
7*a*	Glacier single crystal ||c	75.2	86	4.1
b		63.1	63	3.6
8*a*	Polycrystalline ice	70.7	71	3.6
b		68.6	68	3.6

Linear compressibility θ is determined according to the relationship $\theta = -(1/L_0)(\Delta L_1/\Delta P) + \theta_{Al}$, where L_0 is the distance between the marker strip and the base of the sample, ΔL_1 is the offset of the marker produced at the test pressure, 0.31 kb (ΔP), and θ_{Al} is the linear compressibility of aluminum, taken as 0.43 Mb^{-1}. Fuller explanation of measurements is given in text and Figure 2(*a*, *b*).

*The two marker strips attached to the top of each sample are designated *a* and *b*.

†Symbols ⊥ c and ||c indicate directions perpendicular and parallel to the crystallographic *c* axis, respectively.

TABLE 2. Data on Compressibility of Ice

Data Source	T, °C	Linear Compressibility, Mb^{-1} ⊥c	Linear Compressibility, Mb^{-1} ‖c	Volume Compressibility, Mb^{-1}
Jona and Scherrer [1952]	-16	3.7	3.7	11.1
Bass et al. [1957]	-10	4.1	4.6	12.8
Dantl [1969]	-10	4.0	4.7	12.7
Richards and Speyers [1914]	-7			12
This paper	-10	3.7	3.6	11

offsets registered with ice at the test pressure decreased to 0 when the pressure was reduced to 0 would indicate that the deformation is entirely elastic. All factors considered, the over-all accuracy of the linear compressibility of ice obtained with this technique is about 10%.

Materials tested. Three types of ice were tested: single-crystal ice prepared by the technique of zone refining, single-crystal ice obtained from the terminus of the Mendenhall glacier, Alaska, and polycrystalline ice prepared in the laboratory. The single-crystal specimens were carefully oriented by optical methods to permit measurements of compressibility in directions parallel and perpendicular to the crystallographic *c* axis. All specimens were microscopically flawless; they lacked all trace of bubbles and cracks, appeared completely devoid of small angle boundaries, and contained negligible quantities of impurities, dissolved or solid.

RESULTS

Data are presented in Table 1. Linear compressibility of single crystals is given for the two principal directions, parallel and perpendicular to the crystallographic *c* axis. Small differences between the parallel and perpendicular directions for a particular type of single crystal are probably not too significant when the over-all 10% error in this method is considered. However, both the laboratory-grown crystal and the polycrystalline specimen exhibit a somewhat lower compressibility (3.5–3.6 Mb^{-1}) than the single-crystal ice from the Mendenhall glacier (3.9 Mb^{-1}). If these differences are significant, they could possibly be attributed to the greater elasticity of the naturally occurring ice, which is known to have undergone a long history of annealing recrystallization at the terminus of the Mendenhall glacier.

The mean value of all measurements in the pressure range 0–0.31 kb at $-10°C$ is 3.7 Mb^{-1}. Because of the small difference in compressibility ($<10\%$) between the two principal directions of crystallographic orientation in ice, the volume compressibility of ice should be approximately equal to 3 times the mean linear compressibility, i.e., $3 \times 3.7\ Mb^{-1} = 11.1\ Mb^{-1}$.

Comparisons with published data on the compressibility of ice are shown in Table 2. The data of *Jona and Scherrer* [1952], *Bass et al.* [1957], and *Dantl* [1969] were calculated from measurements of the elastic constants of single-crystal ice; those of *Richards and Speyers* [1914] are a direct determination. Some spread of values is evident. However, the results indicate that the linear compressibilities perpendicular and parallel to the *c* axis agree within 10%, except for Dantl's data, for which the difference is about 18%, and that the volume compressibilities are in good agreement with the direct experimental value of Richards and Speyers and are in disagreement with the measurements of Bridgman.

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